Hardisp.f yet again

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1 Long-period tides

Bos, and Penna, and Dach, Urschl, and Beutler, have found a systematic error in the version of `hardisp` that was distributed in 2006: comparisons with the older IERS routine `ARG`, and with GPS data, show that the sign of the predicted long-period tides in `hardisp` is opposite from both. This has been attributed to a sign error introduced when the amplitudes of the harmonics were changed from those given by Cartwright and Edden (1973) to those given by Hartmann and Wenzel (1995) [hereafter HW]. However, this could not be correct, since the harmonic amplitudes were kept in the normalization of Cartwright and Tayler (1971) [hereafter CT]; the reason for using the HW values was that they applied to a more current epoch.

Considerable testing shows that this explanation was partly correct. No errors were found in the program except for the sign given to the amplitudes of the long-period tides. The problem turns out to be similar to that described by Bos *et al.* (2000): what is the correct phase for the long-period tides? There is universal agreement that an increase of the tidal potential, corresponding to upward motion of the potential height, should be reckoned positive. Following Cartwright and Edden (1973) the potential height can be written as

$$\frac{V}{g} = \sum_{n,m} c_{nm}(t) Y_{nm}(\theta, \phi) \tag{1}$$

where the c_{nm} are functions of time t, and the Y_{nm} are functions of colatitude θ and longitude ϕ.

The sign of the c's will depend, then, on the form of the Y's. However, `hardisp` uses the c's in a slightly different way: it scales them by the admittance between the potential and the load tide, and uses these scaled values to to predict the time series of the tide. In this usage, we cannot arbitrarily change the signs of the c's: rather, we must take them to have the signs they have been assigned according to standard methods of tidal analysis—so the sign convention depends on the type of Y chosen for these standard methods. For harmonic analysis the standard is Doodson (1921), following Darwin (1883b). Darwin actually recognized the possibility of confusion for the sign of the long-period tides, since he wrote:

> The terms in the $\frac{1}{3}(\mathtt{X}^2 + \mathtt{Y}^2 - 2\mathtt{Z}^2)$ function[1] require special consideration. The function of the latitude being $\frac{1}{3} - \sin^2\lambda$, it follows that when in the northern hemisphere it is high-water north of a certain critical latitude, it is low water on the opposite side of that parallel; and the same is true of the southern hemisphere. ... It will be best to adopt a uniform system for the whole earth, and to regard high-tide and high-water as consentaneous in the equatorial belt, and of opposite meanings outside the critical latitudes. In this Report we conceive the function always to be written $\frac{1}{3} - \sin^2\lambda$, so that outside the critical latitudes high-tide is low-water.

Here λ is the latitude, so the relevant Y is positive on the Equator; as this is the opposite of the CT normalization, it implies that the long-period CT tidal amplitudes should be multiplied by -1; or, what comes to the same thing, the phases should have π added to them. This has been done in subroutine `admint`.

2 Precision

Bos and Penna also converted all real variables to double precision. My own preference is to use double precision only as needed—partly to flag to future users when it is in fact required. But I recognize that these choices are somewhat a matter of taste. To test the effects of double precision, I computed 10 years of vertical load tides at Kodiak (152.4972°W, 57.7400°N), using values computed by Scherneck in 1999 from the CSR4 model; this location has large diurnal and semidiurnal tides. The versions of the program that were used were

1. The fully double-precision version of the program produced by Bos and Penna.
2. The (largely) single-precision version distributed in 2006, with the phase of the long-period tides corrected.
3. The same as 2. but with the harmonic amplitudes given to 6 decimals instead of 5.
4. The same as 2. but with the harmonic amplitudes given to 6 decimals instead of 5, and a lower cutoff level, so that a total of 342 harmonics are used, rather than 141.

[1] The X, Y, Z are solid spherical harmonics—DCA.

The size of the tides, and of the various differences, are

What	Std. deviation	Min	Max
Tide	33.317	−83.230	95.190
2-1	0.0014	−0.0065	0.0055
3-2	0.0010	−0.0038	0.0037
4-2	0.0847	−0.4669	0.3699

where all the values are in millimeters. It is clear that adding precision, either to the arithmetic or to the tidal amplitudes, has a negligible effect. Adding more harmonics decreases the error to sub-mm levels, and this seems like an acceptable tradeoff against speed.

3 Other changes

In order to (I hope) make the program more transparent, and in the long run easier to maintain, I have declared all variables, to make their typing explicit. I have also, in most of the routines, used more modern constructions (`enddo` and `if-then`) for control flow, to make the program conform better to the tenets of structured programming. While `enddo` is not part of the Fortran77 standard, it seems to be supported by all current compilers. I have also removed some items that were left over from other uses of the codes, but that are not used in `hardisp`.

References

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