Example application of the IAU 2000 resolutions concerning Earth orientation and rotation

Patrick Wallace[1]

(Original version 20 July 2004, revised 29 July 2004; this reformatted and corrected version 20 January 2006)

Here is an example application of the IAU 2000 resolutions concerning Earth orientation and rotation. The objective is to predict the topocentric apparent direction of a star.

The nomenclature scheme I am using is shown in Fig. 1. Coordinates are in boxes, linked by the various transformations and corrections.

The circumstances are as follows:

- Site $9^\circ\!.712156$ E, $52^\circ\!.385639$ N, 200 m above sea level (ITRS)
- Fictitious *Tycho 2* star, epoch 2000:
  - $[\,\alpha,\delta\,] = 353^\circ\!.22987757,\ +52^\circ\!.27730247$
  - Proper motions: $\mu_\alpha \cos\delta = +22.9$ mas/year, $\mu_\delta = -2.1$ mas/year
  - Parallax 23 mas
  - Radial velocity +25 km/s
- UTC 2003/08/26 00:37:38.973810
- IERS δX, δY, DUT1 corrections: +0.038, −0.118 mas, −0.349535 s
- IERS polar motion $x_p = +0''\!.259371$, $y_p = +0''\!.415573$

The results, in summary form, are as follows:

ICRS epoch 2000 $[\,\alpha,\delta\,]$	$353^\circ\!.22987757000$	$+52^\circ\!.27730247000$
BCRS $[\,\alpha,\delta\,]$	$353^\circ\!.22991549972$	$+52^\circ\!.27730034185$
astrometric $[\,\alpha,\delta\,]$	$353^\circ\!.22991889091$	$+52^\circ\!.27730584235$
GCRS $[\,\alpha,\delta\,]$	$353^\circ\!.23789320667$	$+52^\circ\!.27695262534$
CIRS $[\,\alpha,\delta\,]$	$353^\circ\!.23300208264$	$+52^\circ\!.29554173960$
topocentric $[\,h,\delta\,]$	$-0^\circ\!.29507962185$	$+52^\circ\!.29549062657$
topocentric $[\,Az, Alt\,]$	$116^\circ\!.44983979538$	$+89^\circ\!.79843387822$
observed $[\,Az, Alt\,]$	$116^\circ\!.44983979538$	$+89^\circ\!.79848801461$

BCRS stands for baryentric celestial reference system. GCRS stands for geocentric celestial reference system. CIRS stands for celestial intermediate reference system; this $[\,\alpha,\delta\,]$ is the IAU 2000 counterpart to geocentric apparent place, and is referred to the celestial intermediate origin (CIO) instead of the equinox. The coordinates $[\,h,\delta\,]$ and $[\,Az, Alt\,]$ are not affected by the introduction of the IAU 2000 methods.

[1]HMNAO/CCLRC/RAL, p.t.wallace@rl.ac.uk

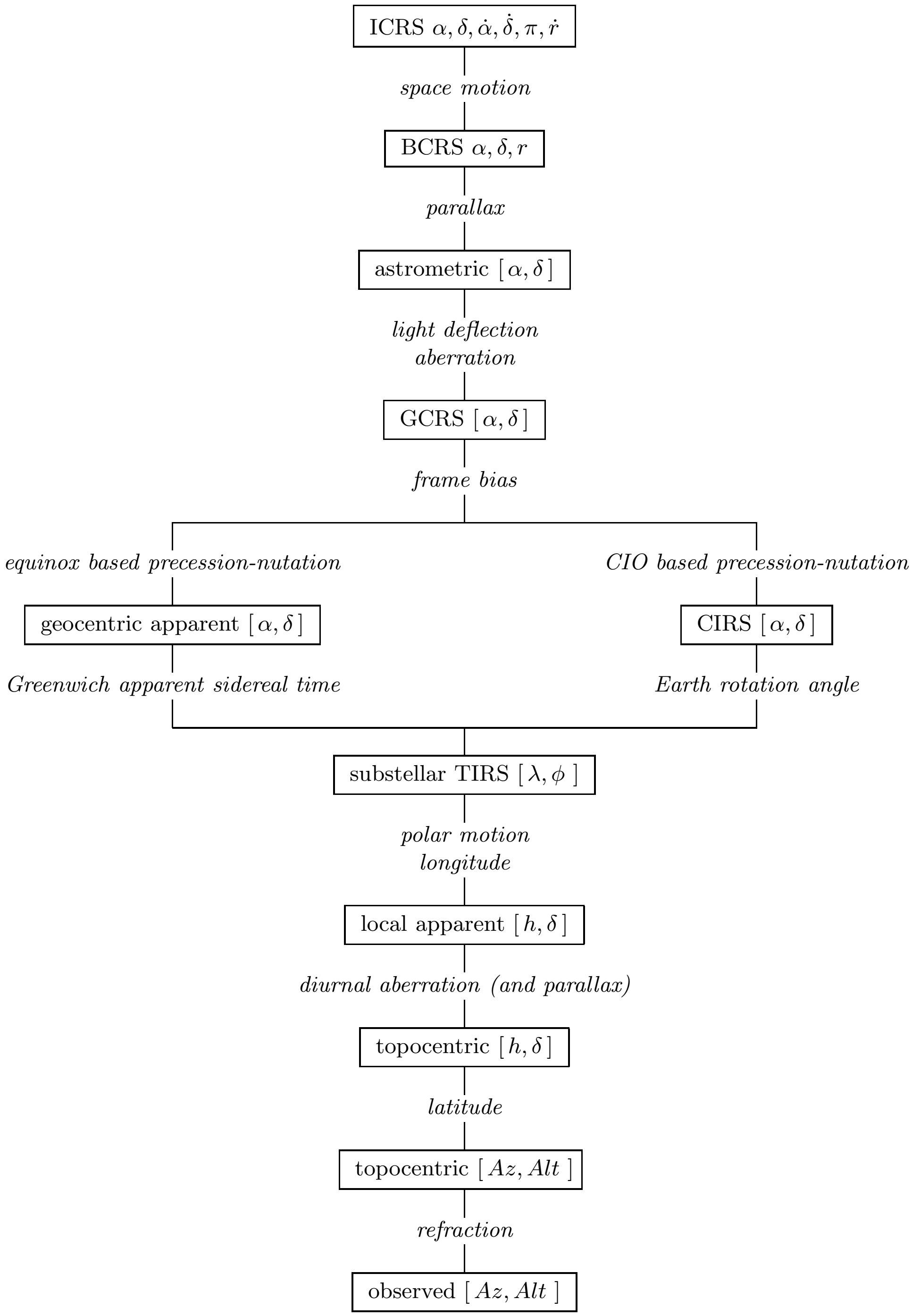

Figure 1: **Relationship between celestial coordinates**

Since the introduction of first the ICRS and then the IAU 2000 resolutions, the moving equator and ecliptic no longer provide the basis of high-precision astrometry, and mean places have fallen out of use. ICRS was nominally aligned to mean J2000, with a residual *frame bias* of about 23 mas that can be ignored for many applications. In addition, a new zero-point for right ascension of date, the CIO, has replaced the equinox, breaking the final link with the ecliptic as far as the user is concerned. The figure relates a star's ICRS $[\alpha, \delta]$ to the actual line-of-sight to the star. The right-hand branch shows the modern CIO/ERA method of reckoning the Earth rotation; the classical equinox/GST method, shown in the left-hand branch, remains an option for supporting older applications.

The key variables and some of the workings are presented below.

Before we apply the IAU 2000 celestial-to-terrestrial transformation, we must first estimate the geocentric direction of the star by applying the usual corrections for proper motion, light deflection and annual aberration.

For the example date:

TT 52877.0268884005800000 (MJD)

we will use the following (approximate) Earth ephemeris data:

heliocentric posn	+0.895306712607	−0.430362177777	−0.186583142292	AU
barycentric posn	+0.898130398596	−0.433663195906	−0.188058184682	AU
barycentric vel	+0.007714484109	+0.013933051305	+0.006040258850	AU/day

The ICRS RA,Dec at the catalog epoch (2000.0) is:

$[\alpha, \delta]$ ICRS epoch 2000 $353^{\circ}\!.22987757000$ $+52^{\circ}\!.27730247000$

Applying space motion and parallax, we obtain the astrometric place – the ICRS coordinates of an infinitely distant star coincident with the one under study:

astrometric place $353^{\circ}\!.22991889091$ $+52^{\circ}\!.27730584235$

The light deflection from the Sun (we will neglect the other solar-system bodies in this example) takes us to:

with light deflection $353^{\circ}\!.22991848163$ $+52^{\circ}\!.27730517509$

Annual aberration produces the proper direction, which is the GCRS $[\alpha, \delta]$:

with aberration $353^{\circ}\!.23789320667$ $+52^{\circ}\!.27695262534$

This completes the preliminaries; we are now ready to apply the IAU 2000 celestial-to-terrestrial transformations that are the purpose of this demonstration.

To predict the orientation of the Earth we can either implement the IAU resolutions literally, working via classical methods, or we can use the direct series supplied by IERS. The former is somewhat more computationally efficient (and more instructive), the latter easier to get right.

To demonstrate the classical methods, we start with the frame bias:

$d\psi$ −41.7750 mas
$d\epsilon$ −6.8192 mas
$d\alpha$ −14.6000 mas

giving an ICRS to mean J2000 rotation matrix of:

+0.99999999999999420000	−0.00000007078279744199	+0.00000008056217146976
+0.00000007078279477857	+0.99999999999999690000	+0.00000003306041454222
−0.00000008056217380987	−0.00000003306040883981	+0.99999999999999620000

The IAU 2000 corrections to the IAU 1976 precession, for the current date, are:

$d\Delta\psi$	-10.932010 mas
$d\Delta\epsilon$	-0.920821 mas

giving an obliquity of date of:

ϵ_A $+84379''.739145661$

and ultimately a J2000-to-date classical precession matrix:

+0.99999960442692650000	−0.00081577397935781730	−0.00035448385722876160
+0.00081577398094000060	+0.99999966725634080000	−0.00000014012603875794
+0.00035448385358768210	−0.00000014905272408423	+0.99999993717058570000

Using the IAU 2000A model, we obtain the classical nutation components:

	$\Delta\psi$	$\Delta\epsilon$
luni-solar	$-12''.687774156$	$+5''.669802082$
planetary	$+0''.000048676$	$+0''.000119415$

which combine to give:

nutation, IAU 2000A	$-12''.687725480$	$+5''.669921497$

and the following classical mean-to-true nutation matrix:

+0.99999999810814740000	+0.00005643620233914664	+0.00002446753280028101
−0.00005643552974070033	+0.99999999802968630000	−0.00002748924554327810
−0.00002446908414069593	+0.00002748786465307690	+0.99999999932284070000

Combining the rotation matrices, we obtain for the classical N×P×B matrix:

+0.99999965722043850000	−0.00075940856976379120	−0.00032993579616347590
+0.00075939951242126470	+0.99999971127592400000	−0.00002757624279218965
+0.00032995664253816620	+0.00002732568025683818	+0.99999994519095910000

This matrix could be used to transform the geocentric place into the true place of date, with the CIP as the pole and the true equinox of date as the α origin. The transformation to terrestrial coordinates would then require the mean sidereal time and the equation of the equinoxes, complicated functions involving both UT and TT. As we are following IAU 2000 methods, we are instead going to work via the celestial intermediate system, so that the transformation to terrestrial coordinates requires only the Earth rotation angle, a simple linear function of UT.

We extract the CIP X, Y coordinates from the matrix, elements (3,1) and (3,2):

CIP X, Y	+0.000329956642538	+0.000027325680257

(As mentioned earlier we can, if we wish, generate X, Y directly from series. For the date in question we obtain:

from series	+0.000329956644592	+0.000027325684592

Note the good agreement.)

If the utmost accuracy is needed, the next step is to add the small IERS δX, δY corrections (which come from VLBI observations):

IAU 2000A	+0.000329956642538	+0.000027325680257
corrected	+0.000329956826767	+0.000027325108177

Having obtained X, Y, we compute the small quantity s using the series for $s + XY/2$:

s -2.900355 mas

From X, Y and s we can obtain the rotation matrix that transforms directions from the GCRS into the system of date, CIRS:

$$\begin{array}{rrr} +0.99999994556424450000 & +0.00000000955326444293 & -0.00032995682715159210 \\ -0.00000001856936992539 & +0.99999999962666920000 & -0.00002732510353706674 \\ +0.00032995682676736510 & +0.00002732510817669448 & +0.99999994519091400000 \end{array}$$

Earlier, we computed the $[\alpha, \delta]$ at which the star appears, in the geocentric celestial reference system:

GCRS	$353\overset{\circ}{.}23789320667$	$+52\overset{\circ}{.}27695262534$

The rotation matrix transforms this into the celestial intermediate reference system:

CIRS	$353\overset{\circ}{.}23300208264$	$+52\overset{\circ}{.}29554173960$

We are now ready to move from coordinates on the celestial sphere into coordinates on the Earth. This involves Earth rotation, together with three small effects:

diurnal aberration	$<< 1$ arcsec
s'	$<< 0.1$ mas
polar motion	$<< 1$ arcsec

Because these effects are small, the precise way they are applied is of little consequence; individual applications thus have some leeway. For example, in some cases it may be more convenient to deal with the diurnal aberration at the same time as annual aberration and to eliminate the geocentric stage completely. In a similar way, some applications will need the star direction in terms of the geographical coordinates of the sub-star point whereas others (and this example) will express the same information in the form of a local hour angle and declination.

In all cases, the Earth rotation angle is the main element in the transformation. The UT1 is:

UT1 52877.02614148466 (MJD)

and the corresponding ERA is:

ERA	$343^{\circ}\!.2256920994647$

The IERS Conventions set out the CIRS-to-ITRS transformation as a large rotation about the z-axis corresponding to the ERA, followed by small z, y and x rotations that take into account s' and polar motion.

The tiny quantity s' for the given date is:

s'	-0.001714681 mas

The rotation matrix[2] for the s' and polar motion portion is:

$$\begin{array}{rrr} +0.99999999999920940000 & -0.00000000000831300878 & +0.00000125746609283028 \\ +0.00000000001084649458 & +0.99999999999797040000 & -0.00000201475475899438 \\ -0.00000125746609281098 & +0.00000201475475900642 & +0.99999999999717980000 \end{array}$$

Allowing for diurnal aberration, Earth rotation, site longitude and polar motion produces the following local $[\,h, \delta\,]$:

$[\,h, \delta\,]$ topocentric	$-0^{\circ}\!.29507962185$	$+52^{\circ}\!.29549062657$

or, rotated into local horizon coordinates:

$[\,Az, Alt\,]$ topocentric	$116^{\circ}\!.44983979538$	$+89^{\circ}\!.79843387822$

For this site and time, the star has just passed almost overhead. Note that this is the "topocentric" rather than "observed" position; if pointing a real telescope or antenna, the next correction would be for atmospheric refraction.

Note that some numbers are quoted to a number of decimal places beyond that corresponding to the floating-point precision of the computer used to generate them. Repeating the calculations on a different platform may produce slightly different results.

[2] The earlier editions of this document displayed the matrix elements in column order rather than row, so that the transpose of the correct matrix was given. This mistake was reported by Steffen Höfler of the Technische Universität Dresden.