4 Terrestrial reference systems and frames (complete chapter update as of 01 April 2019)

4.1 Concepts and terminology

4.1.1 Basic concepts

Terrestrial Reference Systems and their realizations. A Terrestrial Reference System (TRS) is a spatial reference system co-rotating with the Earth in its diurnal motion in space. In such a system, positions of points attached to the solid surface of the Earth have coordinates which undergo only small variations with time, due to geophysical effects (*e.g.* tectonic or tidal deformations). In the physical model adopted in astrogeodesy, a TRS is modeled as a reference trihedron close to the Earth and co-rotating with it. In the Newtonian framework, the physical space is considered as an Euclidian affine space of dimension 3. In this case, such a reference trihedron is an Euclidian affine frame (O, E). O is a point of the space named **origin** and E is a basis of the associated vector space. The currently adopted restrictions on E are to be right-handed, orthogonal with the same length for the basis vectors. The triplet of unit vectors collinear to the basis vectors expresses the **orientation** of the TRS and the common length of these vectors its **scale**,

$$\lambda = \|\vec{E}_i\|_i \qquad i = 1, 2, 3. \tag{4.1}$$

Here, we consider geocentric TRSs for which the origin is close to the Earth's center of mass (geocenter), the orientation is equatorial (the Z axis is the direction of the pole, and X axis is the prime meridian) and the scale is close to the SI meter. In addition to Cartesian coordinates (naturally associated with such a TRS), other coordinate systems, *e.g.* geographical coordinates, could be used. For a general reference on coordinate systems, see Boucher (2001).

Relationship between different TRSs. Under these hypotheses, the general transformation of the Cartesian coordinates of any point close to the Earth from TRS (1) to TRS (2), represented by their coordinate vectors $\vec{X}^{(1)}$ and $\vec{X}^{(2)}$, respectively, is given by a three-dimensional similarity ($\vec{T}_{1,2}$ is a translation vector, $\lambda_{1,2}$ a scale factor and $R_{1,2}$ a rotation matrix)

$$\vec{X}^{(2)} = \vec{T}_{1,2} + \lambda_{1,2} \cdot R_{1,2} \cdot \vec{X}^{(1)}. \tag{4.2}$$

This concept can be generalized in the frame of a relativistic background model such as Einstein's General Relativity, using the spatial part of a local Cartesian coordinate system (Boucher, 1986). For more details concerning general relativistic models, see Chapters 10 and 11.

In the application of equation (4.2), the IERS uses the linearized formulas. The standard transformation between two reference systems is an Euclidian similarity of seven parameters: three translation components, one scale factor, and three rotation angles, designated respectively, $T1$, $T2$, $T3$, D, $R1$, $R2$, $R3$, and their first time derivatives: $\dot{T}1$, $\dot{T}2$, $\dot{T}3$, $\dot{D}$, $\dot{R}1$, $\dot{R}2$, $\dot{R}3$. The transformation of a coordinate vector $\vec{X}_1$, expressed in reference system (1), into a coordinate vector $\vec{X}_2$, expressed in reference system (2), is given by

$$\vec{X}_2 = \vec{X}_1 + \vec{\mathcal{T}} + D\vec{X}_1 + \mathcal{R}\vec{X}_1, \tag{4.3}$$

where $\vec{\mathcal{T}} = \vec{T}_{1,2}$, $D = \lambda_{1,2} - 1$, $\mathcal{R} = (R_{1,2} - I)$, and I is the identity matrix so that

$$\mathcal{T} = \begin{pmatrix} T1 \\ T2 \\ T3 \end{pmatrix}, \quad \mathcal{R} = \begin{pmatrix} 0 & -R3 & R2 \\ R3 & 0 & -R1 \\ -R2 & R1 & 0 \end{pmatrix}.$$

It is assumed that equation (4.3) is linear for sets of station coordinates provided by space geodesy techniques. Origin differences are at the few centimeter level, and differences in scale and orientation are at the level of a few parts in 10^{-9}. Generally, $\vec{X}_1, \vec{X}_2, \mathcal{T}, D$ and $\mathcal{R}$ are functions of time. Differentiating equation (4.3) with respect to time gives

$$\dot{\vec{X}}_2 = \dot{\vec{X}}_1 + \dot{\vec{\mathcal{T}}} + \dot{D}\vec{X}_1 + D\dot{\vec{X}}_1 + \dot{\mathcal{R}}\vec{X}_1 + \mathcal{R}\dot{\vec{X}}_1. \tag{4.4}$$

D and $\mathcal{R}$ are at the level of 10^{-9} and $\dot{\vec{X}}$ is about 10 cm per year, so the terms $D\dot{\vec{X}}_1$ and $\mathcal{R}\dot{\vec{X}}_1$ which represent about 0.0001 mm over 100 years are negligible. Therefore, equation (4.4) could be written as

$$\dot{\vec{X}}_2 = \dot{\vec{X}}_1 + \dot{\vec{\mathcal{T}}} + \dot{D}\vec{X}_1 + \dot{\mathcal{R}}\vec{X}_1. \tag{4.5}$$

It is fundamental to distinguish between a TRS, having a theoretical definition, and its realization, a Terrestrial Reference Frame (TRF), to which users have access.

Terrestrial Reference Frame (TRF). A Terrestrial Reference Frame is defined as the realization of a TRS, through the specification of its origin, orientation axes and scale, and their time evolution. We consider here that the realization is achieved by a set of physical points with precisely determined coordinates in a specific coordinate system as a realization of a Terrestrial Reference System. It is also designated as a crust-based TRF when physical points are anchored to the Earth's surface.

4.1.2 Crust-based TRF

Crust-based TRFs are those currently determined within the IERS activities, either by analysis centers or by combination centers, and ultimately as IERS products (see Section 4.1.5).

The coordinates of the TRF points are hereafter referred to as regularized coordinates. The general model connecting the instantaneous position of a point anchored on the Earth's crust at epoch t, $\vec{X}(t)$, and a regularized position $\vec{X}_R(t)$ is

$$\vec{X}(t) = \vec{X}_R(t) + \sum_i \Delta\vec{X}_i(t). \tag{4.6}$$

The purpose of the introduction of a regularized position is to remove high-frequency time variations using conventional corrections $\Delta\vec{X}_i(t)$, in order to obtain a position with more regular time variation.

It is essential that the same conventional models be adopted and used by all analysis centers dealing with space geodesy data. The currently adopted models for site displacements are described in Chapter 7.

4.1.3 Space geodesy TRF solutions

TRFs computed by analysis centers within the IERS activities are determined mainly from space geodetic observations, and are provided as solutions. A solution is defined as a set of coordinates with their covariance information (variance-covariance matrix) or an equivalent form (normal equations).

Seven parameters are needed to define a TRF at a given epoch, to which are added their time derivatives to define the TRF time evolution. The selection of the 14 parameters, establishes the TRF origin, scale, orientation and their time evolution.

Space geodesy techniques are not sensitive to all the parameters of the TRF definition. The origin is theoretically accessible through dynamical techniques (LLR, SLR, GNSS, DORIS), being the center of mass (point around which the satellites orbit). The scale depends on some physical parameters (*e.g.* geo-gravitational constant GM and speed of light c) and relativistic modeling, and is also subject to technique systematic errors. The orientation of the TRF, unobservable by any technique, is arbitrary or conventionally defined.

Since space geodesy observations do not contain all the necessary information to completely establish a TRF, some additional information is needed to complete the TRF definition. In terms of normal equations, usually constructed upon space geodesy observations, this situation is reflected by the fact that the normal matrix, N, is singular, since it has a rank deficiency corresponding to the number of frame parameters which are not reduced by the observations.

The general form of the singular normal equation constructed upon space geodesy observations could be written as

$$N(\Delta\vec{X}) = K, \tag{4.7}$$

where $\Delta\vec{X} = \vec{X} - \vec{X}_0$ designates the linearized unknowns, $\vec{X}_0$ the vector of *a priori* parameters, and K is the right-hand side of the normal equation.

In order to cope with this rank deficiency, the analysis centers currently add one of the following constraints upon all or a sub-set of stations:

1. Removable constraints: solutions for which the estimated station positions and/or velocities are constrained to external values within an uncertainty $\sigma \approx 10^{-5}$ m for positions and m/y for velocities. This type of constraint is easily removable, see for instance Altamimi *et al.* (2002a; 2002b).
2. Loose constraints: solutions where the uncertainty applied to the constraints is $\sigma \geq 1$ m for positions and ≥ 10 cm/y for velocities.
3. Minimum constraints used solely to define the TRF using a minimum amount of required information. For more details on the concepts and practical use of minimum constraints, see for instance Sillard and Boucher (2001) and Altamimi *et al.* (2002a).

Note that the method where very tight constraints ($\sigma \leq 10^{-10}$ m) are applied (which are numerically not easy to remove), is no longer suitable and may alter the real quality of the estimated parameters.

In case of removable or loose constraints, the reference frame constraint is equivalent to adding the following pseudo-observation equation

$$\vec{X} - \vec{X}_0 = 0, \tag{4.8}$$

where $\vec{X}$ is the vector of estimated parameters (positions and/or velocities) and $\vec{X}_0$ is that of the *a priori* parameters.

In the case of minimum constraints, the added equation is of the form

$$B(\vec{X} - \vec{X}_0) = 0, \tag{4.9}$$

where $B = (A^TA)^{-1}A^T$ and A is the design matrix of partial derivatives, constructed upon *a priori* values $\vec{X}_0$ given by either

$$A = \begin{pmatrix} . & . & . & . & . & . & . \\ 1 & 0 & 0 & x_0^i & 0 & z_0^i & -y_0^i \\ 0 & 1 & 0 & y_0^i & -z_0^i & 0 & x_0^i \\ 0 & 0 & 1 & z_0^i & y_0^i & -x_0^i & 0 \\ . & . & . & . & . & . & . \end{pmatrix} \tag{4.10}$$

when solving for only station positions, or

$$A = \begin{pmatrix} . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ 1 & 0 & 0 & x_0^i & 0 & z_0^i & -y_0^i & & & & & & & \\ 0 & 1 & 0 & y_0^i & -z_0^i & 0 & x_0^i & & & & 0 & & & \\ 0 & 0 & 1 & z_0^i & y_0^i & -x_0^i & 0 & & & & & & & \\ & & & & & & & 1 & 0 & 0 & x_0^i & 0 & z_0^i & -y_0^i \\ & & & \approx 0 & & & & 0 & 1 & 0 & y_0^i & -z_0^i & 0 & x_0^i \\ & & & & & & & 0 & 0 & 1 & z_0^i & y_0^i & -x_0^i & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . \end{pmatrix} \tag{4.11}$$

when solving for station positions and velocities.

The fundamental distinction between the two approaches is that in equation (4.8), we force $\vec{X}$ to be equal to $\vec{X}_0$ (to within a given σ), while in equation (4.9) we express $\vec{X}$ in the same TRF as $\vec{X}_0$ using the projector B containing the necessary information defining the underlying TRF. Note that the two approaches are sensitive to the configuration and quality of the subset of stations ($\vec{X}_0$) used in these constraints.

In terms of normal equations, equation (4.9) could be written as

$$B^T \Sigma_\theta^{-1} B(\vec{X} - \vec{X}_0) = 0, \tag{4.12}$$

where Σ_θ is a diagonal matrix containing small variances for each of the transformation parameters.

Adding equation (4.12) to the normal equation (4.7) allows it to be inverted and simultaneously to express the estimated solution $\vec{X}$ in the same TRF as the *a priori* solution $\vec{X}_0$. Note that the 7 columns of the design matrix A correspond to the 7 TRF parameters (3 translations, 1 scale factor and 3 rotations). Therefore, the matrix B should be reduced to those parameters which need to be defined when determining a TRF(*e.g.* 3 rotations in almost all techniques and 3 translations in case of VLBI). Similar minimum constraint equations exist to define the time-evolution of a TRF which include station velocities, also composed of a maximum of 7 pseudo-observations.

For more practical details, see, for instance, Altamimi *et al.* (2002a).

4.1.4 The International Terrestrial Reference System

The IERS is in charge of defining, realizing and promoting the International Terrestrial Reference System (ITRS). The ITRS was adopted formally by the IUGG at its General Assembly in Perugia (2007), through its Resolution 2.

To summarize IAG and IUGG resolutions of 1991 and 2007, which are consistent with latest IAU Resolutions:

- The term "conventional terrestrial reference system" (CTRS) is to be used to designate a generic TRS specified by a list of conventional rules fixing its origin scale and orientation.
- The term "geocentric terrestrial reference system" (GTRS) designates a CTRS with origin at the geocenter, considered for the whole Earth system including the oceans and atmosphere.
- The timescale associated with the GTRS is Geocentric Coordinate Time (TCG), and the scale of the spatial coordinates is consistent with TCG.
- The time evolution of the orientation of GTRS follows a no-net-rotation (NNR) condition with regards to the Earth surface.

The spatial definition of the ITRS is consistent with the IUGG 2007 statement, and all practical applications (mapping, navigation) consider ITRS as a three-dimensional system. The practical procedure originally adopted by the IERS ensures that the ITRS orientation coincides with the previous BIH system at the epoch 1984.0.

ITRS definition fulfills the following conditions:

1. It is geocentric, its origin being the center of mass for the whole Earth, including oceans and atmosphere;
2. The unit of length is the meter (SI). The scale is consistent with the TCG time coordinate for a geocentric local frame, in agreement with IAU and IUGG (1991) resolutions. This is obtained by appropriate relativistic modeling;
3. Its orientation was initially given by the Bureau International de l'Heure (BIH) orientation at 1984.0;
4. The time evolution of the orientation is ensured by using a no-net-rotation condition with regards to horizontal tectonic motions over the whole Earth surface.

4.1.5 Realizations of the ITRS

Primary realizations of the ITRS are called International Terrestrial Reference Frame (ITRF) and are produced by the ITRS Center of the IERS. Thirteen versions of the ITRF have been produced, starting with ITRF88 and ending with ITRF2014. Up to the ITRF2000 solution, long-term global solutions (comprising station positions and velocities) from the four techniques (VLBI, SLR, GPS and DORIS) were used as input for the ITRF generation. As described in more detail later, starting with the ITRF2005, time series of station positions and Earth Orientation Parameters (EOPs) have been used as input data for the ITRF construction.

The regularized coordinates of ITRF stations are parametric functions of time. They are composed of:

- station positions at a reference epoch and station velocities until ITRF2008. Note that station velocities were taken from geophysical models before ITRF91. See Section 4.2.2.
- station positions at a reference epoch, station velocities and post-seismic deformation (PSD) functions for some stations in ITRF2014. See Section 4.2.5.

The mathematical model used for the ITRF combination is described extensively in ITRF publications (see the bibliography section), *e.g.* Altamimi *et al.* (2002b, 2007, 2011, 2016).

The main equations of the combination model involve a 14-parameter similarity transformation, station positions, velocities and EOPs, written as:

$$\left\{ \begin{array}{lcl} \vec{X}_s^i - \delta\vec{X}_{PSD}^i(t_s) & = & \vec{X}_c^i + (t_s^i - t_0)\dot{\vec{X}}_c^i + \Delta\vec{X}_f^i(t_s^i) \\ & + & \vec{T}_k + D_k\vec{X}_c^i + R_k\vec{X}_c^i + (t_s^i - t_k)\left[\dot{\vec{T}}_k + \dot{D}_k\vec{X}_c^i + \dot{R}_k\vec{X}_c^i\right], \\ \dot{\vec{X}}_s^i & = & \dot{\vec{X}}_c^i + \dot{\vec{T}}_k + \dot{D}_k\vec{X}_c^i + \dot{R}_k\vec{X}_c^i, \end{array} \right. \tag{4.13}$$

As they represent a key element

$$\left\{ \begin{array}{lcl} x_s^p & = & x_c^p + R2_k, \\ y_s^p & = & y_c^p + R1_k, \\ UT_s & = & UT_c - \frac{1}{f}R3_k, \\ \dot{x}_s^p & = & \dot{x}_c^p, \\ \dot{y}_s^p & = & \dot{y}_c^p, \\ LOD_s & = & LOD_c, \end{array} \right. \tag{4.14}$$

where for each point i, $\vec{X}_s^i$ (at epoch t_s^i) and $\dot{\vec{X}}_s^i$ are positions and velocities of technique solution s and $\vec{X}_c^i$ (at epoch t_0) and $\dot{\vec{X}}_c^i$ are those of the combined solution c. For each individual frame k, as implicitly defined by solution s, D_k is the scale factor, $\vec{T}_k$ the translation vector and R_k the rotation matrix. The dotted parameters designate their derivatives with respect to time. The translation vector $\vec{T}_k$ is composed of three origin components, namely T_x, T_y, T_z, and the rotation matrix of three small rotation angles: R_x, R_y, R_z, following the three axes, respectively X, Y, Z. t_k is a conventionally selected epoch of the 7 transformation parameters. In addition to equation (4.13) involving station positions (and velocities), the EOPs are included by equation (4.14), making use of pole coordinates x_s^p, y_s^p and Universal Time UT_s as well as their daily rates $\dot{x}_s^p$, $\dot{y}_s^p$ and LOD_s. The two terms $\delta\vec{X}_{PSD}^i(t_s^i)$ and $\Delta\vec{X}_f^i(t_s^i)$ are described below.

The link between the combined frame and the EOPs is ensured via the three rotation parameters appearing in the first three lines of equation (4.14).

Moreover, a precise definition of the reference frame associated with the resulting long-term solution (comprising station positions at a reference epoch, station velocities and EOPs) has to be clearly specified. As transformation parameters are estimated between each weekly (or session-wise) frame and the long-term frame, it becomes necessary to define the long-term frame origin, scale, orientation and their time evolution, and therefore to complete the rank deficiency of the constructed normal equation. It is essential that the long-term solutions be representative of the mean origin and mean scale information of the space geodesy techniques. The particular type of minimal constraints introduced in Altamimi *et al.* (2007) have been designed for such purpose and are used to preserve the long-term solution origin (for SLR and DORIS) and scale (for VLBI, SLR and DORIS).

The current procedure adopted for the ITRF formation involves two steps (Altamimi *et al.*, 2002a, 2007, 2011, 2016): (1) stacking the individual time series to estimate a long-term solution per technique comprising station positions at a reference epoch, station velocities, and daily EOPs, and (2) combining the resulting long-term solutions of the four techniques together with the local ties at co-location sites. During the first step, nearby stations or multiple segments of the same station in case of discontinuities are constrained to have the same velocity. In the second step, co-located stations are constrained to have the same velocities, by choosing one station per technique.

The first line of equation (4.13) and the entire equation (4.14) are used to estimate long-term solutions for each technique, by accumulating (rigorously stacking) the individual technique time series of station positions and EOPs. In this process, the second line of equation (4.13) and the rates of the translation, scale and rotation parameters are not included, because station velocities are not available on a weekly (daily) basis.

In the most recent ITRF release, namely ITRF2014, two terms have been introduced in equation (4.13) when accumulating station position time series in order to estimate more consistent coordinates and velocities between space geodetic techniques at co-location sites. The terms $\Delta\vec{X}^i_f(t)$ added to the right-hand side of equation (4.13) first accounts for periodic variations in station coordinates which are not handled by displacement models of Chapter 7. The general equation used for the modeling of the periodic signals embedded in the time series of station positions is written as:

$$\Delta\vec{X}_f(t) = \sum_{j=1}^{n_f} \mathbf{a}^j \cos(\omega_j t) + \mathbf{b}^j \sin(\omega_j t), \tag{4.15}$$

where $\Delta\vec{X}_f(t)$ is the total sum of the contributions of all the frequencies considered, n_f is the number of frequencies, $\omega_j = \frac{2\pi}{\tau_j}$, where τ_j is the period of the j^{th} frequency, *e.g.* annual and semi-annual in ITRF2014 (station index i has been removed for clarity but a^j and b^j are different for every station). Each frequency adds six parameters per station, i.e $(a^j_x, a^j_y, a^j_z, b^j_x, b^j_y, b^j_z)^T$, in addition to the six position and velocity parameters. However, although they have been estimated, these parameters are not part of the ITRF2014 regularized coordinate model.

The second term that has been added in equation 4.13 is the post-seismic deformation model, $\delta\vec{X}^i_{PSD}(t)$, for stations affected by major earthquakes. It is provided in the form of logarithmic or/and exponential functions that have been fitted beforehand in station position time series. See Section 4.2.5. These displacement models are not part of Chapter 7 but are part of ITRF2014 regularized coordinates since they have to be added to the ITRF2014 linear model. See Section 4.2.5.

The combination method makes use of local ties at co-location sites where two or more geodetic techniques are operated. The local ties are used as additional observations with proper variances. They are usually derived from local surveys using either classical geodesy or the global navigation satellite systems (GNSS). As they represent a key element of the ITRF combination, they should be at least as accurate, if not more accurate, than the individual space geodesy solutions incorporated in the ITRF combination.

Currently, ITRF solutions are published by the ITRS Center in IERS *Technical Notes* (*cf.* Altamimi *et al.*, 2017), and also in peer-reviewed articles (Altamimi *et al.*, 2002b, 2007, 2011, 2016). The number following the designation “ITRF” specifies the last year whose data were used for the formation of the frame. Hence, ITRF2014 designates the frame of station positions and velocities constructed in 2015 using data available until the end of 2014 (2015.12 for GNSS).

4.2 ITRF products

4.2.1 The IERS network

The IERS network was initially defined through all tracking instruments used by the various individual analysis centers contributing to the IERS. All SLR, LLR and VLBI systems were included. Progressively, GNSS stations from the IGS were added as well as the DORIS tracking network. The network also included, from its beginning, a selection of ground markers, specifically those used for mobile equipment and those currently included in local surveys performed to monitor local eccentricities between instruments for co-location sites or for site stability checks.

Each point is currently identified by the assignment of a DOMES (Directory of MERIT sites) number. The explanation of the DOMES numbering system is given below. Close points are clustered into one site. The current rule is that all points which could be linked by a co-location survey (up to 30 km for historical sites) should be included in the IERS network as a unique site having a unique DOMES site number. In reality, for a local tie to be precise at the 1 mm level, the extension of a co-location site should not exceed 1 km.

The concept of co-location can be defined as the situation where two or more instruments are occupying simultaneously or subsequently very close locations that are very precisely surveyed in three dimensions. These include situations such as simultaneous or non-simultaneous measurements and instruments of the same or different techniques. Usually co-located points should belong to a unique IERS site.

4.2.2 History of ITRF products

The history of the ITRF goes back to 1984, when for the first time a combined TRF (called BIH Terrestrial System 1984 BTS84), was established using station coordinates derived from VLBI, LLR, SLR and Doppler/TRANSIT (the predecessor of GPS) observations (Boucher and Altamimi, 1985). BTS84 was realized in the framework of the activities of the BIH, being a coordinating center for the international MERIT project (Monitoring of Earth Rotation and Intercomparison of Techniques; Wilkins, 2000). Three other successive BTS realizations were then achieved, ending with BTS87, when in 1988, the IERS was created by the IUGG and the IAU.

Until the time of writing, thirteen versions of the ITRF were published, starting with ITRF88 and ending with ITRF2014, each of which superseded its predecessor.

From ITRF88 till ITRF93, the ITRF specification can be summarized as follows:

- Origin and scale: defined by an average of selected SLR solutions;
- Orientation: defined by successive alignment since BTS87 whose orientation was aligned to the BIH EOP series. Note that the ITRF93 orientation and its rate were again realigned to the IERS EOP series;
- Orientation time evolution: No global velocity field was estimated for ITRF88, ITRF89 and ITRF90, so the AM0-2 model of Minster and Jordan (1978) was recommended. Starting with ITRF91 and till ITRF93, combined velocity fields were estimated. The ITRF91 orientation rate was aligned to that of the NNR-NUVEL-1 model (Argus and Gordon, 1991), and ITRF92 to NNR-NUVEL-1A, adapted from NNR-NUVEL-1 according to DeMets *et al.* (1994), while ITRF93 was aligned to the IERS EOP series.

Since the ITRF94, full variance matrices of the individual solutions incorporated in the ITRF combination have been used. At that time, the ITRF94 definition was achieved as follows (Boucher *et al.*, 1996):

- Origin: defined by a weighted mean of selected SLR and GPS solutions;
- Scale: defined by a weighted mean of VLBI, SLR and GPS solutions, corrected by 0.7 ppb to meet the IUGG and IAU requirement to be compatible with TCG, while analysis centers provide solutions that are compatible with TT (Terrestrial Time);
- Orientation: aligned to the ITRF92;

- Orientation time evolution: velocity field aligned to the model NNR-NUVEL-1A, using the 7 rates of the transformation parameters.

The ITRF96 was then aligned to the ITRF94, and the ITRF97 to the ITRF96 using the 14 transformation parameters (Boucher *et al.*, 1998; 1999).

The ITRF2000 was intended to be a standard solution for geo-referencing and all Earth science applications. Therefore, in addition to primary core stations observed by VLBI, LLR, SLR, GPS and DORIS, the ITRF2000 was densified by regional GPS networks in Alaska, Antarctica, Asia, Europe, North and South America and the Pacific.

The individual solutions used in the ITRF2000 combination were generated by the IERS analysis centers using removable, loose or minimum constraints.

In terms of reference frame definition, the ITRF2000 is characterized by the following properties:

- the scale is realized by setting to zero the scale and scale rate parameters between ITRF2000 and a weighted average of VLBI and the most consistent SLR solutions. Unlike the ITRF97 scale which is compatible with TCG, that of the ITRF2000 is compatible with TT;
- the origin is realized by setting to zero the translation components and their rates between ITRF2000 and a weighted average of the most consistent SLR solutions;
- the orientation is aligned to that of the ITRF97 at 1997.0 and its rate is aligned, conventionally, to that of the geological model NNR-NUVEL-1A (Argus and Gordon, 1991; DeMets *et al.*, 1990; 1994).

4.2.3 ITRF2005

For the first time in the ITRF history, the ITRF2005 used as input data time series of station positions (weekly from satellite techniques and 24-hour session-wise from VLBI) and daily EOPs. One set of time series per space geodesy technique was considered as input to the ITRF2005 combination. These solutions are the official time series provided by the international services of the 4 techniques, known as Technique Centers (TC) within the IERS. Note that these official TC solutions result from a combination at the weekly (daily) basis of the corresponding individual solutions provided by the analysis centers participating in the activities of each TC. Official time series were submitted to the ITRF2005 by the International VLBI Service (IVS), the International Laser Ranging Service (ILRS) and the International GNSS Service (IGS). At the time of the ITRF2005 release, official weekly combined solutions from the International DORIS Service (IDS) were not available, so that individual solutions were submitted by two DORIS analysis centers. For more details the reader may refer to Altamimi *et al.* (2007).

The ITRF2005 generation consists of two steps: (1) stacking the individual time series to estimate a long-term solution per technique comprising station positions at a reference epoch and velocities as well as daily EOPs; and (2) combining the resulting long-term solutions of the four techniques together with the local ties in co-location sites. Therefore, in addition to the usual ITRF products (station positions and velocities), other important ITRF2005 results are also available to the users, namely:

1. full ITRF2005 and per technique SINEX files containing station positions, velocities and EOPs with complete variance-covariance matrices;
2. time series of station position residuals resulting from the stacking of the individual time series of the 4 techniques;
3. geocenter time series from SLR and DORIS. There is no useful geocenter motion information from GPS/IGS, the submitted weekly solutions being aligned to ITRF2000;
4. full time series of EOPs consistent with the ITRF2005.

The ITRF2005 origin was defined in such a way that it had zero translations and translation rates with respect to the Earth center of mass, averaged by the SLR time series spanning 13 years of observations. Its scale was defined by nullifying the scale and its rate with respect to the VLBI time series spanning 26 years of observations. It should be noted that after the release of the ITRF2005 it was discovered that the IVS VLBI solutions used for the ITRF2005 construction did not include pole tide corrections referenced to the mean pole path recommended by the IERS Conventions 2003. Post-ITRF2005 analyses of IVS solutions where the mean pole tide correction was applied reveals a constant scale offset of -0.5 ppb with respect to the IVS solutions used in ITRF2005 (Altamimi and Collilieux, 2008). The ITRF2005 orientation (at epoch 2000.0) and its rate are aligned to the ITRF2000 using 70 stations of high geodetic quality.

4.2.4 ITRF2008

Following the same strategy initiated with the ITRF2005 release, the ITRF2008 was a refined solution based on reprocessed solutions of the four space geodesy techniques: VLBI, SLR, GPS and DORIS, spanning 29, 26, 12.5 and 16 years of observations, respectively.

The ITRF2008 was composed of 934 stations located at 580 sites with an imbalanced distribution between the northern (463 sites) and the southern hemisphere (117 sites).

There were in total 105 co-location sites, 91 of these have local ties available for the ITRF2008 combination. Note that, unfortunately, not all these co-located instruments are currently operating. For instance, among the 6 sites having 4 techniques, only two are currently operating: Hartebeesthoek, South Africa and Greenbelt, MD, USA.

The ITRF2008 is specified by the following frame parameters:

- Origin: The ITRF2008 origin was defined in such a way that there are null translation parameters at epoch 2005.0 and null translation rate with respect to the ILRS SLR time series.
- Scale: The scale of the ITRF2008 was defined in such a way that there are null scale factor at epoch 2005.0 and null scale rate with respect to the mean scale and scale rate of VLBI and SLR time series.
- Orientation: The ITRF2008 orientation was defined in such a way that there are null rotation parameters at epoch 2005.0 and null rotation rates between ITRF2008 and ITRF2005. These two conditions were applied over a set of 179 reference stations located at 131 sites, including 107 GPS, 27 VLBI, 15 SLR and 12 DORIS sites.

4.2.5 ITRF2014, the current reference realization of the ITRS

For the first time in the history of ITRF, the ITRF2014 is generated with an enhanced modeling of nonlinear station motions, including seasonal (annual and semi-annual) signals of station positions and post-seismic deformation (PSD) for sites that were subject to major earthquakes.

The seasonal signals were modeled using sinusoidal functions with annual and semi-annual frequencies as described by equation (4.15), while the PSDs were accounted for, before the stacking, by applying parametric models that were first fitted to IGS daily station position time series. Corrections, predicted by the GNSS fitted models to the nearby stations of the three other techniques (DORIS, SLR and VLBI) at earthquake co-location sites, were then applied before stacking their respective time series. The main motivation of modelling the periodic (annual and semi-annual) signals is to ensure the most robust estimation of site linear velocities, and so they are not part of the ITRF2014 products. On the contrary, the fitted PSD parametric models are effectively part of the ITRF2014 products and the users should be aware of their importance and usage, depending on their applications. Failing to do so could introduce position errors at the decimeter level for many stations impacted by PSDs.

The user should recall that by definition, and by construction, the ITRF is a linear (secular) frame and so the supplied station positions vary, with the station velocities provided, as a piece-wise function of time, even for earthquake sites that have significant post-seismic deformation. However, in case of ITRF2014 and for stations subject to PSD, the user should add the sum of all PSD corrections to the linearly propagated position, using the following equation:

$$\vec{X}_{PSD}(t) = \vec{X}(t_0) + \dot{\vec{X}}.(t - t_0) + \delta\vec{X}_{PSD}(t), \quad (4.16)$$

where $\delta\vec{X}_{PSD}(t)$ is the total sum of PSD corrections at epoch t. The ITRF2014 PSD parametric models, together with all equations allowing users to compute the PSD corrections and Fortran subroutines are available at the ITRF2014 website (http://itrf.ign.fr/ITRF_solutions/2014/).

The ITRF2014 is specified by the following frame parameters:

1. Origin: The ITRF2014 origin is defined in such a way that there are zero translation parameters at epoch 2010.0 and zero translation rates with respect to the mean origin of the ILRS SLR time series;

2. Scale: The scale of the ITRF2014 is defined in such a way that there is zero scale factor at epoch 2010.0 and zero scale rate with respect to the average of the implicit scales and scale rates of VLBI and SLR time series;

3. Orientation: The ITRF2014 orientation is defined in such a way that there are zero rotation parameters at epoch 2010.0 and zero rotation rates between ITRF2014 and ITRF2008. These two conditions were applied over a set of 127 reference stations located at 125 sites.

4.2.6 Usage of ITRF station coordinates

The ITRF main products are provided in the form of station positions at a given epoch and constant velocities, as piece-wise linear functions. Parametric models to account for post-seismic deformations for stations subject to major earthquakes are also part of the ITRF products, since ITRF2014. Meanwhile, for various reasons, there are particular cases where users would need to add specific corrections to ITRF coordinates in order to meet their particular applications. The currently identified cases are the following:

A) *Solid Earth tides*

To account for the displacement due to solid Earth tides, analysis centers of all four techniques use a model $\Delta\vec{X}_{tidM}$ that contains a time-independent part, so that the regularized positions obtained are termed "conventional tide-free", according to the nomenclature in the Introduction of the Conventions.

Such a convention has been taken since the first solid Earth tides model of the MERIT Standards. Consequently, the ITRF has adopted the same option and is therefore a "conventional tide free" frame. To adopt a different model, $\Delta\vec{X}_{tid}$, a user would need to apply the following formula to obtain coordinates $\vec{X}$ consistent with this model:

$$\vec{X} = \vec{X}_{ITRF} + (\Delta\vec{X}_{tidM} - \Delta\vec{X}_{tid}). \quad (4.17)$$

For more details concerning tidal corrections, see Chapter 7.

B) *Relativistic scale*

Individual centers of all four techniques use a scale consistent with Terrestrial Time (TT). The ITRF has also adopted this option (except ITRF94, 96 and 97). It should be noted that the ITRS scale is specified to be consistent with Geocentric Coordinate Time (TCG). Consequently, if coordinates $\vec{X}$ consistent with TCG are needed, users need to apply the following formula:

$$\vec{X} = (1 + L_G)\vec{X}_{ITRF}, \quad (4.18)$$

where $L_G = 0.6969290134 \times 10^{-9}$ (IAU Resolution B1.9, 24th IAU General Assembly, Manchester 2000). Note that consistency between numerical constants should be ensured as described in Chapter 1.

C) *Geocentric positions*

The ITRF origin should be considered as the mean Earth Center of Mass, averaged over the time span of the SLR observations used and modeled as a secular (linear) function of time. If an instantaneous geocentric position $\vec{X}$ is required, it should be computed as

$$\vec{X} = \vec{X}_{ITRF} + \Delta\vec{X}_G, \quad (4.19)$$

where $\Delta\vec{X}_G$ represents the origin translation (vector from the instantaneous Center of Mass to the ITRF origin) due to non-secular geocenter motion.

4.2.7 Transformation parameters between ITRF solutions

Table 4.1 lists transformation parameters and their rates from ITRF2014 to previous ITRF versions, which should be used with equations (4.3) and (4.5). The values listed in this table have been compiled from those already published in previous IERS Technical Notes as well as from comparisons between pairs of successive ITRF solutions, from ITRF2000 up to ITRF2014. Moreover, it should be noted that these parameters are adjusted values which are heavily dependent on the weighting as well as the number and distribution of the implied common sites between these frames. Therefore, using different subsets of common stations between two ITRF solutions to estimate transformation parameters would not necessarily yield values consistent with those of Table 4.1.

Table 4.1: Transformation parameters relating the ITRF2014 to past ITRFs. “ppb” refers to parts per billion (or 10^{-9}). The rate units are “per year.”

ITRF Solution	$T1$ (mm)	$T2$ (mm)	$T3$ (mm)	D (ppb)	$R1$ (mas)	$R2$ (mas)	$R3$ (mas)	Epoch
ITRF2008	1.6	1.9	2.4	-0.02	0.00	0.00	0.00	2010.0
rates	0.0	0.0	-0.1	0.03	0.00	0.00	0.00	
ITRF2005	2.6	1.0	-2.3	0.92	0.00	0.00	0.00	2010.0
rates	0.3	0.0	-0.1	0.03	0.00	0.00	0.00	
ITRF2000	0.7	1.2	-26.1	2.12	0.00	0.00	0.00	2010.0
rates	0.1	0.1	-1.9	0.11	0.00	0.00	0.00	
ITRF97	7.4	-0.5	-62.8	3.80	0.00	0.00	0.26	2010.0
rates	0.1	-0.5	-3.3	0.12	0.00	0.00	0.02	
ITRF96	7.4	-0.5	-62.8	3.80	0.00	0.00	0.26	2010.0
rates	0.1	-0.5	-3.3	0.12	0.00	0.00	0.02	
ITRF94	7.4	-0.5	-62.8	3.80	0.00	0.00	0.26	2010.0
rates	0.1	-0.5	-3.3	0.12	0.00	0.00	0.02	
ITRF93	-50.4	3.3	-60.2	4.29	-2.81	-3.38	0.40	2010.0
rates	-2.8	-0.1	-2.5	0.12	-0.11	-0.19	0.07	
ITRF92	15.4	1.5	-70.8	3.09	0.00	0.00	0.26	2010.0
rates	0.1	-0.5	-3.3	0.12	0.00	0.00	0.02	
ITRF91	27.4	15.5	-76.8	4.49	0.00	0.00	0.26	2010.0
rates	0.1	-0.5	-3.3	0.12	0.00	0.00	0.02	
ITRF90	25.4	11.5	-92.8	4.79	0.00	0.00	0.26	2010.0
rates	0.1	-0.5	-3.3	0.12	0.00	0.00	0.02	
ITRF89	30.4	35.5	-130.8	8.19	0.00	0.00	0.26	2010.0
rates	0.1	-0.5	-3.3	0.12	0.00	0.00	0.02	
ITRF88	25.4	-0.5	-154.8	11.29	0.10	0.00	0.26	2010.0
rates	0.1	-0.5	-3.3	0.12	0.00	0.00	0.02	

ITRF solutions are specified by Cartesian equatorial coordinates X, Y and Z. If needed, they can be transformed to geographical coordinates (λ, ϕ, h) referred to an ellipsoid. In this case the

GRS80 ellipsoid is recommended (semi-major axis $a = 6378137.0m$, squared eccentricity $e^2 = 0.00669438002290$). See the IERS Conventions' web page for the subroutine at <[1]>.

4.3 Access to the ITRS

Several ways could be used to express point positions in the ITRS, *e.g.*

- direct use of ITRF station positions and velocities;
- aligning a GNSS solution (*e.g.* obtained from a double-difference processing) to the ITRF using the minimum constraints approach, via the inclusion of known ITRF/IGS stations in the GNSS processing as described in Altamimi and Gross (2017);
- use of the Precise Point Positioning (PPP) method which implies the usage of satellite orbits and clocks corrections (*e.g.* IGS products) that should themselves be expressed in an ITRS realization;
- use of similarity transformation formulas that exist between a particular TRF and an ITRF solution.

All information on ITRF solutions since ITRF94 may be found at <[2]>.

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[1] http://iers-conventions.obspm.fr/ or http://maia.usno.navy.mil/conventions/

[2] http://itrf.ign.fr/ITRF_solutions/index.php

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